

Using Computer Algebra Systems (CAS) to Build Stronger Understanding



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www.calculatorsoftware.co.uk/nspire

This talk will have a....

A Beginning

Background information about me & CAS, and the remit of this talk.

A Middle

CAS activities covering various different maths topics for students aged 12-18 years.

An End

...in 40 minutes' time!

And you can download all that you see today from


www.calculatorsoftware.co.uk/nspire

Curious Questions



CAS ... what it's not!



cas  

- cash converters**
- cash generator**
- casio**
- casualty**

Press Enter to search.



Citizens **A**dvice **S**cotland
Central **A**llocation **S**ystem
CESG **A**ssured **S**ervice
Census **A**rea **S**tatistics
Chief of **A**ir **S**taff
Cinema **A**udio **S**ociety
Computing **A**t **S**chool
Contained **A**ir **S**olutions
Centre of **A**frican **S**tudies
Consulting **A**rborist **S**ociety

Chemical **A**bstracts **S**ervice
Confirmation of **A**cceptance for **S**tudies
Cloud **A**pplications and **S**ecurity
Centre for **A**merican **S**tudies
Court of **A**rbitration for **S**port
Churches **A**gency for **S**afeguarding
Communications **A**dvisory **S**ervice
Centre for **A**tmospheric **S**cience
Circuits **A**nd **S**ystems Research Group
Community **A**ction **S**outhwark



Previous

3 4 5 6 7 8 9 10 11 12

Next

Texas Instruments Nspire CX-CAS Graphic Calculator with ...

[www.amazon.co.uk](#) › ... › [Office Electronics](#) › [Calculators](#) › [Graphing](#) ▾

Texas Instruments Nspire CX-CAS Graphic Calculator with Touchpad: Amazon.co.uk: Office Products.



Previous

4 5 6 7 8 9 10 11 12 13

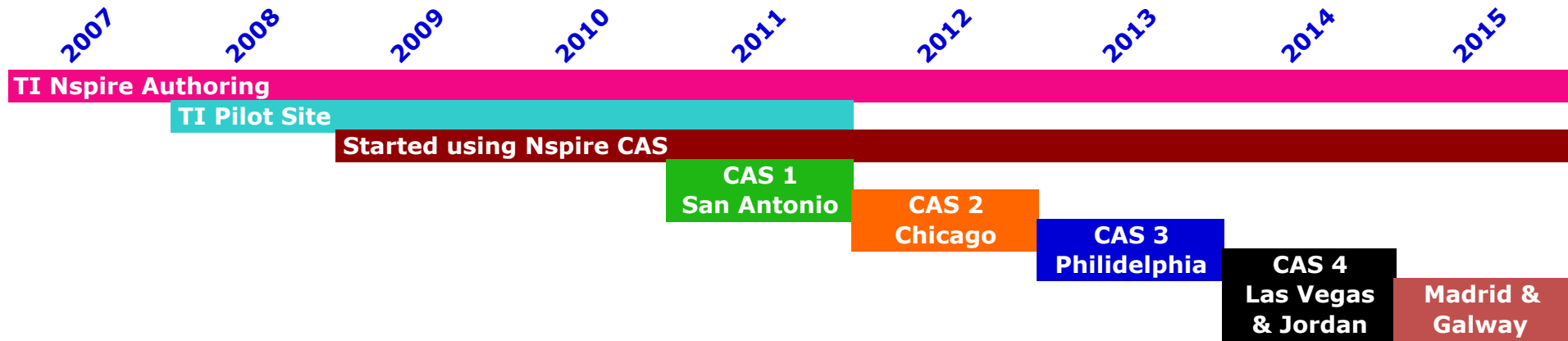
Next

CAS View - GeoGebraWiki

[wiki.geogebra.org/en/CAS_View](#) ▾

The **CAS** View allows you to use GeoGebra's **CAS** (Computer Algebra System) for symbolic computations. It consists of cells with an Input Field at the top and ...

My CAS Timeline

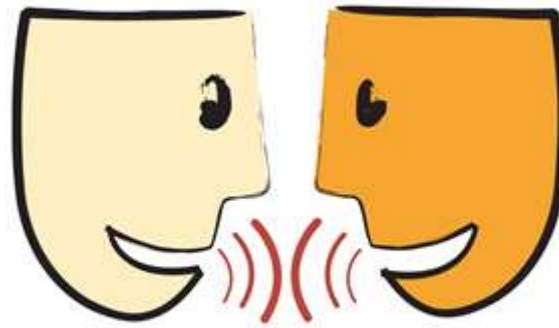


CAS Talks at TI International & European Conferences

- 2011 My first 18 months of CAS usage
- 2012 Trigonometry and Rearranging Equations
- 2013 Linear Equations and Units
- 2014 Extending CAS with functions and programs
- 2015 CAS in Statistics



Not Allowed in Exams



Allowed in Exams



Scottish Exam Arrangements Documents

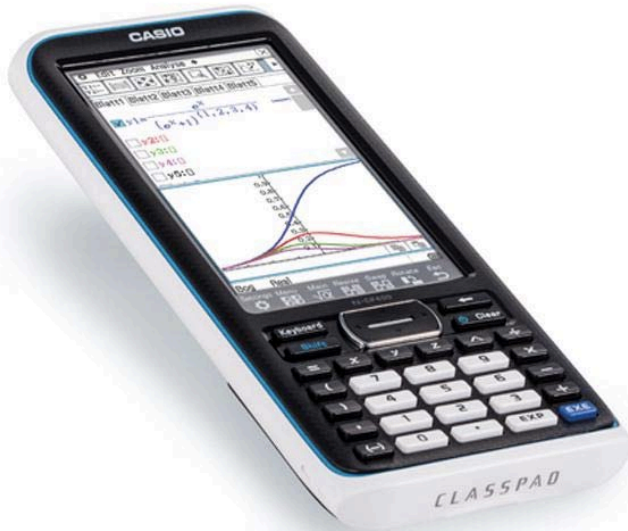
“Calculators with mathematical and graphical facilities and those with **computer algebra systems (CAS)** can be utilised as powerful tools both for processing data, especially in the study of statistics, and **for reinforcing mathematical concepts.**”

“The elementary calculus studied ... is extended to differentiation of sums, products, quotients and composites of elementary functions and to integration using standard results and substitution methods respectively. ...**Computer algebra systems** can be **used extensively for consolidation and extension.**”

A Gamble? An Uphill Struggle?

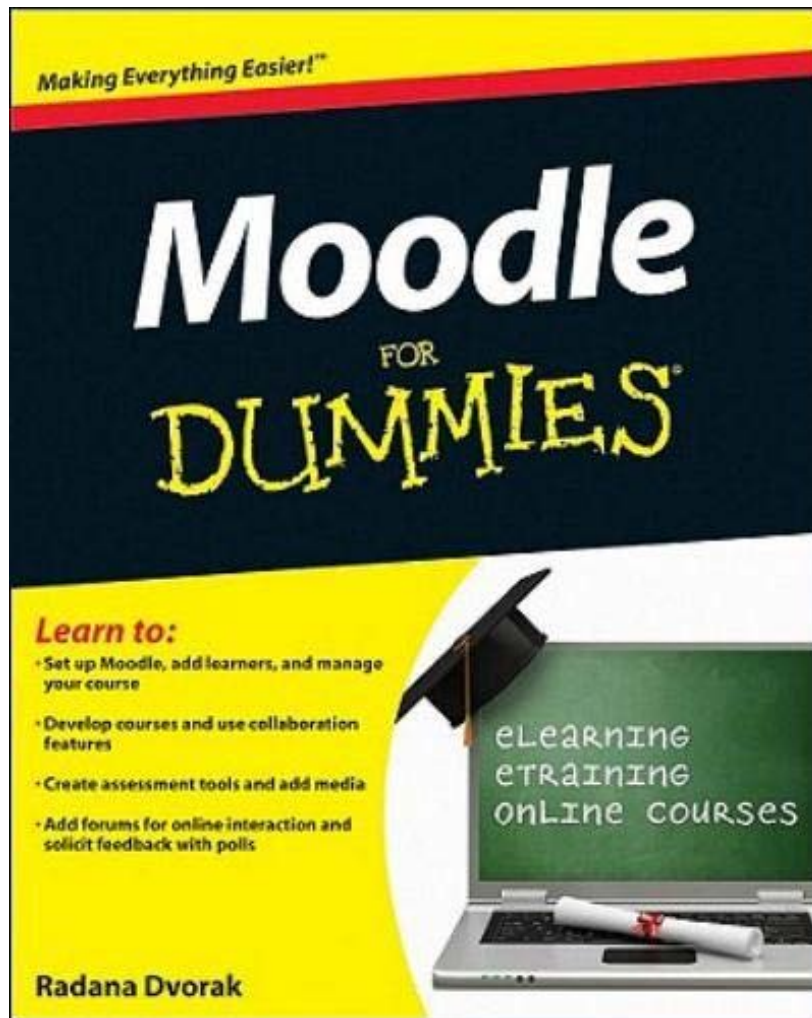


CAS Handheld of Choice

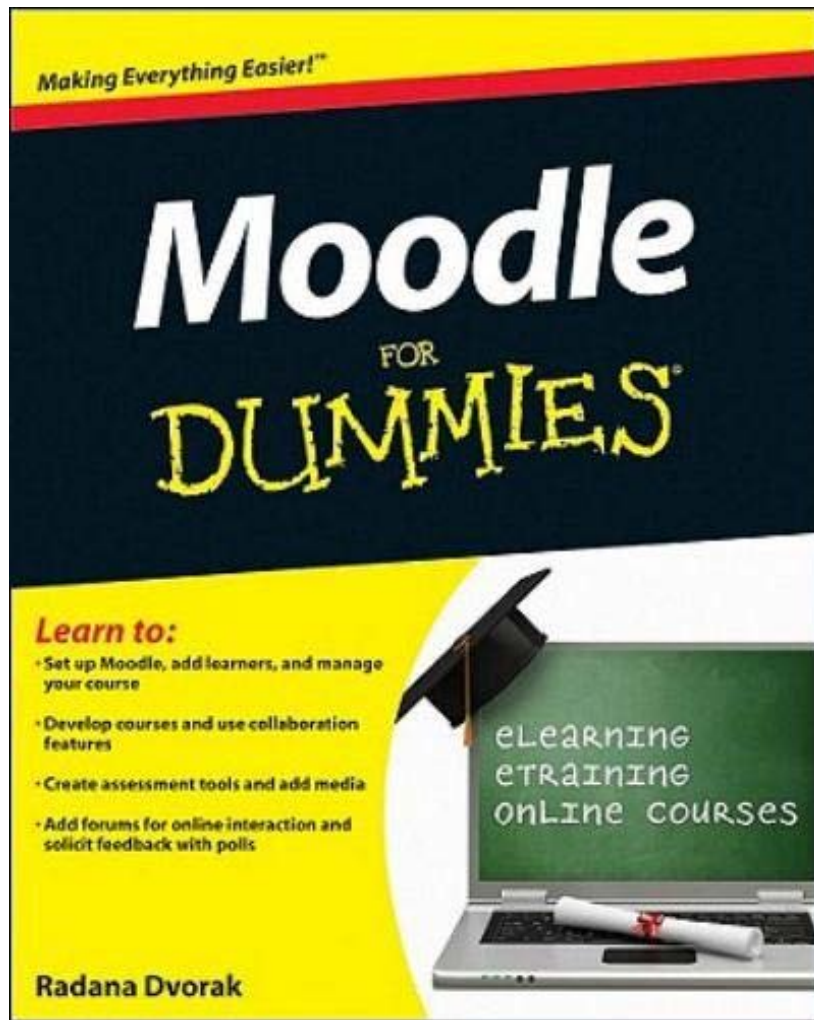


Generation Z (born 1995–2015)

... The learners of this generation are



Generation Z (born 1995–2015)



... The learners of this generation are impatient, seem to expect immediate results, and multitask with tech devices at exceptional speeds. They don't like to read instructions — most jump in and get on with it. Their expectations of technology are demanding. This generation will take to eLearning and will push boundaries.

Generation Z's Compatibility with CAS

- ✓ Are play-oriented.
- ✓ Expect immediate results.
- ✓ Expect information to come to them or accessible at one click.
- ✓ Do not read instructions, especially step-by-step outlines, but jump straight in.
- ✓ Do not process as linearly as previous generations.
- ✓ Are impatient if technology is not quick enough — they find something else to do.
- ✓ Trust the medium.

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- ✓ Are impatient if technology is not quick enough — they find something else to do.
- ✓ Trust the medium.
- ✗ Look at graphics first and access text-based media last.
- ✗ Process things at "twitch speed" (ie "more than 100 images a minute.")
- ✗ Do not stay with tasks as long.
- ✗ Do not expect things to go wrong.



Creating Linear Equations

1.1 1.2 1.3 *Creating Li... CAS

The following pages will give you a **recipe to follow** to create an equation, from a starting point of your choosing.

After following the recipe, try and undo each of the stages, **one at a time**, with the aim of returning to what you started with.

Repeat the recipe a few times, from different starting points, before moving on to the next recipe.

1.1 1.2 1.3 *Creating Li... CAS

1. Start with a letter equalling a number.
eg. **r=7**

2. Multiply this equation by a number eg. **x3**

3. Add a number to both sides eg. **+10**

0/99

randomequation()

$$ax \pm b = \pm d$$

$$Ax \pm b = ax \pm d$$

$$ax \pm b = Ax \pm d$$

$$ax \pm b = cx \pm d$$

$$ax \pm b = d - cx$$

$$b - ax = cx \pm d$$

$$b - ax = d - cx$$

Simultaneous Linear Equations

The screenshot shows a window titled "Sim Equatio... CAS" with three tabs labeled "1.1", "1.2", and "1.3". The content of the window is as follows:

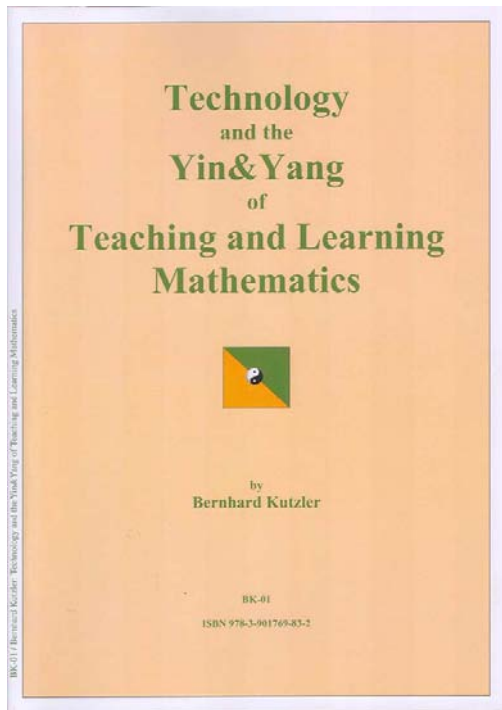
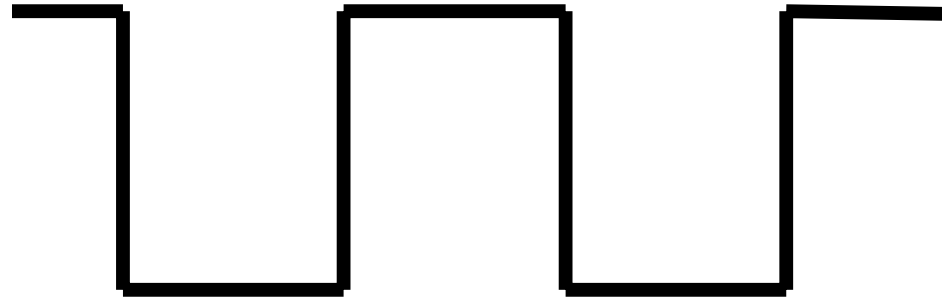
© This page shows an example of its use

$x+2 \cdot y=12$	$x+2 \cdot y=12$
$3 \cdot x-4 \cdot y=15$	$3 \cdot x-4 \cdot y=15$
<i>multiplyby2</i> ($x+2 \cdot y=12$)	$2 \cdot x+4 \cdot y=24$
<i>add</i> ($3 \cdot x-4 \cdot y=15, 2 \cdot x+4 \cdot y=24$)	$5 \cdot x=39$

© ... and then solve this as normal.

[]

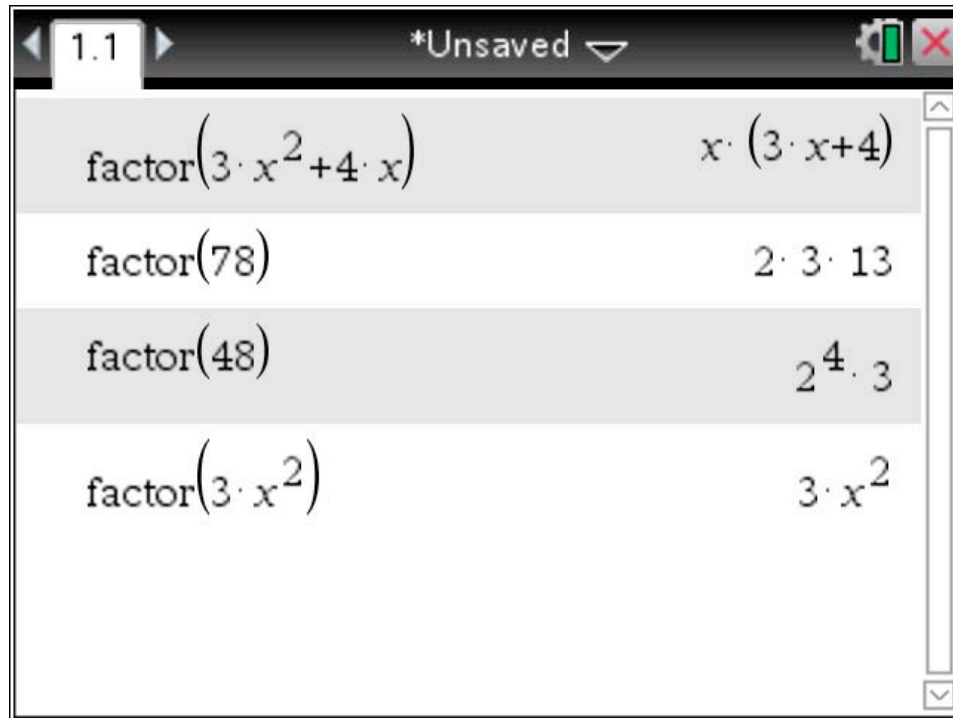
Avoiding Interruptions



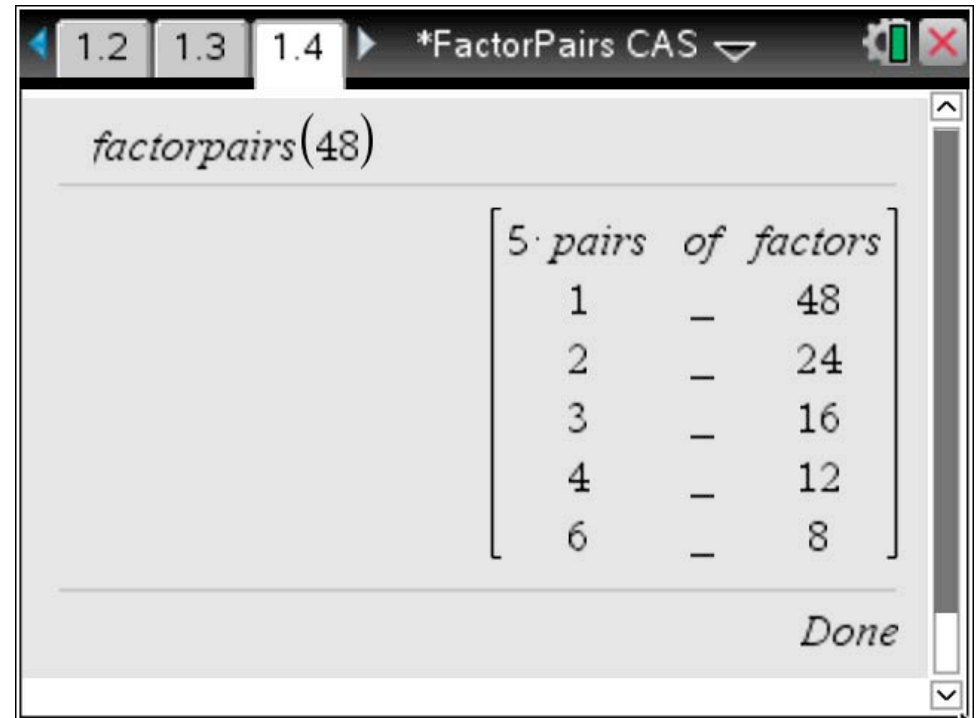
“The picture demonstrates that a student, while trying to learn a new skill, repeatedly has to interrupt the learning process in order to perform a simplification”

Factor Pairs

The Issue



More Helpful



Beneficial when preparing for Factorising expressions

Unexpected Behaviour – 1

The screenshot shows a window titled "1.1" with a status bar indicating "*Unsaved". The window contains a table with two columns and two rows of mathematical expressions. The first row shows $a \cdot r + a \cdot s$ on both sides. The second row shows $v \cdot b + v \cdot c$ on the left and $(b+c) \cdot v$ on the right. A vertical scrollbar is on the right side, and the page number "2/99" is in the bottom right corner.

$a \cdot r + a \cdot s$	$a \cdot r + a \cdot s$
$v \cdot b + v \cdot c$	$(b+c) \cdot v$

2/99

Unexpected Behaviour – 2

The image shows a presentation slide with a title bar and a table of algebraic expressions. The title bar includes a tab labeled '1.1', the text '*Unsaved', and window control icons. The table has three rows, each with a horizontal line below it. The expressions are written in a cursive font. The bottom right corner of the slide displays '3/99'.

$a \cdot r + a \cdot s$	$a \cdot r + a \cdot s$
$v \cdot b + v \cdot c$	$(b + c) \cdot v$
$a \cdot d + a \cdot f$	$a \cdot (d + f)$

3/99

Rearranging Equations

What do we think about when faced with rearranging these formula to make x the subject....

$$kx + m = n \qquad \frac{a}{x} + b = c \qquad r = \frac{x + p}{x - p}$$

I now know why students find rearranging so tricky!

Preparing to use CAS

4.2 4.3 4.4 ▶ *Part

$p+h$ is the same as

- $h+p$
- $-p-h$
- $-(-p-h)$
- $-(h-p)$
- $p-(-h)$

4.3 4.4 4.5 ▶ *Part 1

$a \cdot k + a \cdot p$ is the same as

- $a \cdot (p+k)$
- $a \cdot (k+p)$
- $(p+k) \cdot a$
- $(k+p) \cdot a$
- $a \cdot p + a \cdot k$
- $k \cdot a + p \cdot a$

Preparing to use CAS

◀ 4.1 4.2 4.3 ▶ Part 1 Rea

Dividing x by 2 is the same as

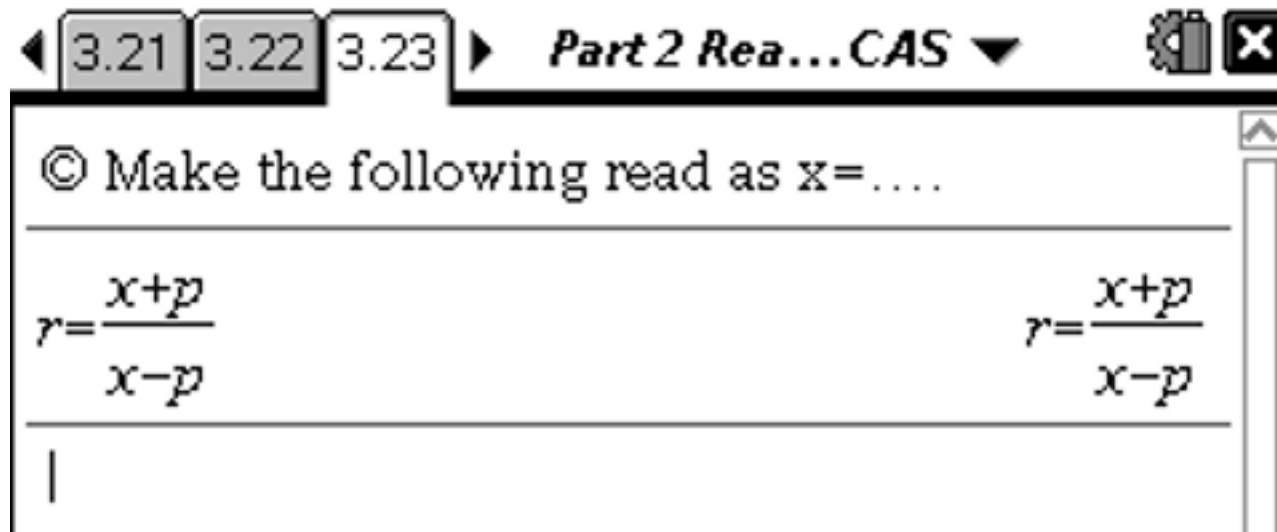
<input type="checkbox"/>	$\frac{1}{2}x$
<input type="checkbox"/>	$\frac{x}{2}$
<input type="checkbox"/>	$x \div 2$
<input type="checkbox"/>	$2x$
<input type="checkbox"/>	$x/2$
<input type="checkbox"/>	$0.5x$
<input type="checkbox"/>	$\frac{2}{x}$
<input type="checkbox"/>	\sqrt{x}

◀ 4.6 4.7 4.8 ▶ Part 1 Rea

$4a+12$ divided by 2 is

<input type="checkbox"/>	$\frac{4a}{2}+12$
<input type="checkbox"/>	$\frac{4a}{2}+\frac{12}{2}$
<input type="checkbox"/>	$4a+\frac{12}{2}$
<input type="checkbox"/>	$\frac{1}{2}(4a+12)$
<input type="checkbox"/>	$(4a+12) \div 2$
<input type="checkbox"/>	$4a+12 \div 2$

Rearranging Formulae



◀ 3.21 3.22 3.23 ▶ Part 2 Rea... CAS ▼ ⚙️ ✕

© Make the following read as $x = \dots$

$r = \frac{x+p}{x-p}$	$r = \frac{x+p}{x-p}$
-----------------------	-----------------------

|

[▶ Video of Elizabeth solving this.](#)

Elizabeth's Jotter

$$18. r = \frac{x+p}{x-p}$$

$$r(x-p) = x+p$$

$$rx-pr = x+p$$

$$rx = x+p+pr \quad rx-pr-x = p$$

$$rx-x = p+pr \quad x(r-x) = p+pr$$

~~$x(r-x) = p+pr$~~

$$x(r+1) = p+pr$$

$$x = \frac{p+pr}{r+1}$$

$$x = \frac{p}{r+1} + \frac{pr}{r+1}$$

$$x = p + \frac{p}{r+1}$$

$$18. r = \frac{x+p}{x-p}$$

$$r(x-p) = x+p$$

$$rx-pr = x+p$$

$$rx-pr-x = p$$

$$rx-x = p+pr$$

$$x(r-1) = p+pr$$

$$x = \frac{p+pr}{r-1} \text{ agree}$$

Elizabeth's Test Homework (2 weeks later)

$$14. \quad R = \frac{rs}{rts}$$

$$R(rts) = rs$$

$$Rrts = rs$$

$$sR = rs - RR$$

$$-rst \quad sR = -Rr$$

$$s(R-r) = -Rr$$

$$s = \frac{-Rr}{r-r}$$

] R-r?

More....

m o m e n t s

Trigonometric Equations

2.5 2.6 2.7 Solving Trig ... CAS

You will now have to use the other commands you've seen when you press the **VAR** button:

inversesine(*equation*)
inversecosine(*equation*)
inversetangent(*equation*)

2.9 2.10 2.11 *Solving Tri... CAS

© Press ENTER, then make the following read as x=....

$\tan(x)=0.247$	$\tan(x)=0.247$
$\text{inversetangent}(\tan(x)=0.247)$	$x=13.874$

3/99

Further ... Quicker

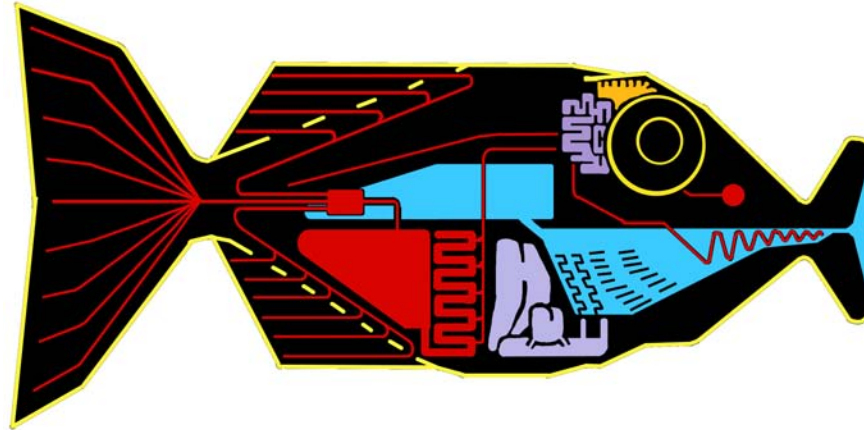
Feedback Messages

The screenshot shows a TI-84 Plus calculator interface with the following elements:

- Navigation buttons: 2.20, 2.21, 2.22 (highlighted), and a right arrow.
- Window title: *Solving Tri... CAS
- Instruction: © Press ENTER, then make the following read as x=....
- Equation 1: $5 \cdot \sin(x) - 4 = 0.$
- Equation 2: $5 \cdot \sin(x) - 4 = 0.$
- Equation 3: $(5 \cdot \sin(x) - 4 = 0.) + 4$
- Equation 4: $5 \cdot \sin(x) = 4.$
- Equation 5: $\text{inversesine}(5 \cdot \sin(x) = 4.)$
- Feedback message: "Inverse sine not appropriate"
- Page number: 4/99

Babel Fish*

**Verbal
Decisions**



**Language of
Mathematics**

* **Hitchhiker's Guide to the Galaxy, Douglas Adams, 1981**

Mathematics is....

Mathematics is....

Problem solving by reasoning

Mathematics is....

Problem solving by reasoning

Modelling

Operating

Interpreting

Reasoning

Mathematics is....

Problem solving by reasoning

Modelling

Operating

Interpreting

Reasoning

Doing → Planning

The Sine Rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

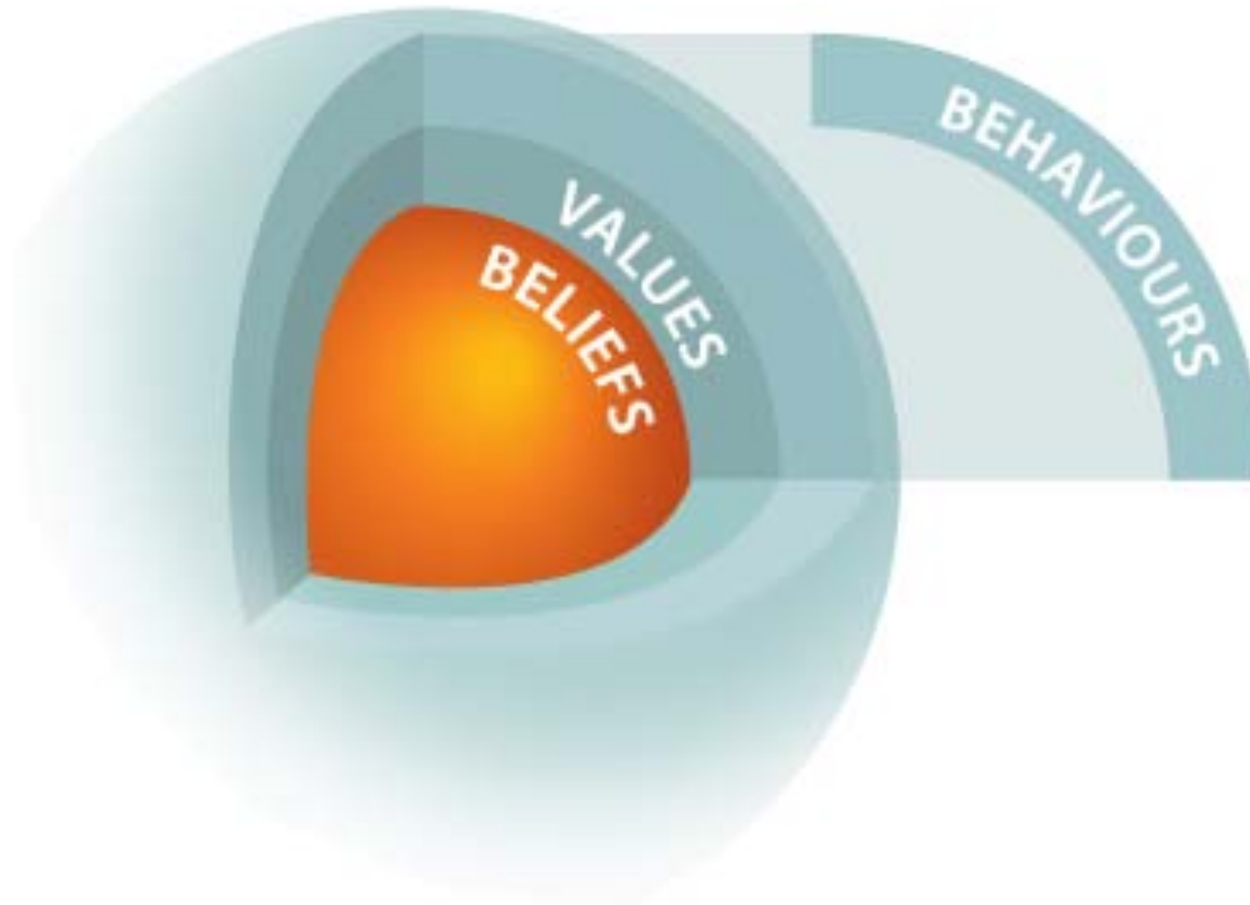


'The Lesson that Crashed'

Why do **I** use CAS?



Why would **you** use CAS?



CAS vs Established Approaches

Are traditional approaches as successful as you think?

CAS vs Established Approaches

**Are traditional approaches as
successful as you think?**

**Or are they mainly successful
on specially designed tasks?**

CAS vs Established Approaches

Are traditional approaches as successful as you think?

Or are they mainly successful on specially designed tasks?

**Do established methods
deserve their exalted status?**

Curious Questions?



Want Copies of Everything?
www.CalculatorSoftware.co.uk/nspire

Thank you for your attention.
Have a great Symposium!

Nevil Hopley

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CAS User